Reliability of Physical Measurements
(Error Analysis)

When we ask the questions: how accurate is a certain measurement? or, how precisely do we know a certain quantity?, we are asking about the error in the value. By the word error we do not mean "mistake," but rather the uncertainty in a quantity caused by all the contributing factors. Every measurement has an error. Every calculated quantity which is based on measured values has an associated error.

The error may be expressed as the absolute error or the relative error. The absolute error is expressed in the units of the measured quantity. The relative error is expressed either as a fraction of the measured quantity or as a percentage of the measured quantity. It is customary to indicate the uncertainty of a quantity as an appendage to the measured value, for example,

$$23.1 \text{ cm} \pm 1\%$$
or
$$23.1 \text{ cm} \pm 0.2 \text{ cm}$$

These are, respectively, the percentage and absolute error.

COMPARING TWO QUANTITIES
The error in the calculated result of an experiment can be as important as the answer itself. Experiments will often involve determining a quantity by two different methods and then seeing if the results agree. It will rarely be, and then only by chance, that there will be exact numerical agreement, so the errors in the two measurements are then compared. Then, if the ranges of the two values overlap, they must be said to agree within the limit of error of that experiment. If they do not overlap, then the two values do not agree.

We cannot say that 19.9 is the same as 20.5, but if these two numbers each have an error such as 19.9 ± 0.3 and 20.5 ± 0.4, then the first value is in the range 19.6 to 20.2 and the second value is in the range 20.1 to 20.9. These ranges overlap, so it may be said that these numbers agree within their errors.

PRECISION AND ACCURACY
The three figures below illustrate the terms accuracy and precision. The center of the target represents the location of an accurate measurement, and each X represents an individual measurement.

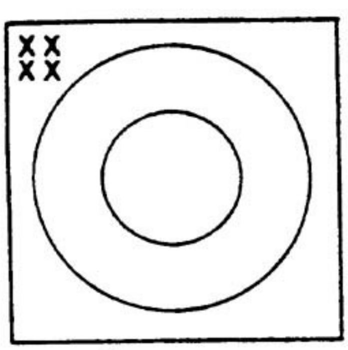
Figure 1.1 good precision, poor accuracy

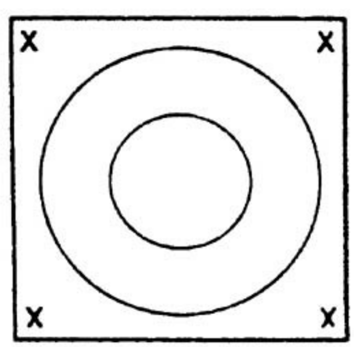
Figure 1.2 poor precision, good accuracy

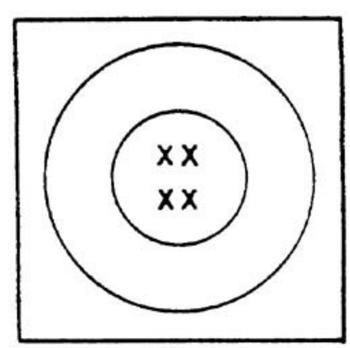
Figure 1.3 good accuracy, good precision

A scientist will repeat a given measurement as often as is practical **to learn the precision** of the measurement, **to improve its accuracy**, or for both reasons. A research scientist, measuring a quantity for the first time, does not know its true value, and thus must estimate through some process the value of the error.

Reliability of Physical Measurements

Significant figures in individual measurements

Let us suppose that the observer recorded 342.6 mm as the length measurement of a bar. The first three figures are regarded as certain because the bar covered slightly more than 342 of the mm marks on the meter stick. The 6 is called a doubtful figure because it was estimated by the observer. The normal procedure followed is to record all of the certain figures and one doubtful figure.

Classification of Error Types: This is not a complete discussion

Instrumental Errors – Limit on accuracy!

Instrument manufacturers normally guarantee that their instruments will have “errors” of not more than a certain amount. Any error statement should not claim better accuracy than this amount.

Observational Errors – Limit accuracy and precision

Observational errors, as the name implies, are caused by limitations of the observer. Adequate training, careful attention, team checking, and comparison of one observer with another are standard means for reducing or eliminating observational errors.

Accidental (or random) Errors – Limit precision

Independent measurements will fluctuate in a nearly random manner. Statistical methods are used to approximate the extent of random errors. The question to be answered is, **“If I measured this quantity again, what value would I get?”**

Standard Error - Precision of the Measurement

The standard error or “error in the mean” is the best guess based upon statistic analysis of the individual measured values that another equivalent measurement would fall into a range of values. This error is derived from the scatter in data points and does not always include the instrumental error. A large number of measurements allow for a calculation of the mean (or average) value and an estimate of the uncertainty in that value. The more measurements one makes, the better one understands the range of variation.

Statistical analysis shows that the precision of the mean varies as the square root of the number of determinations used in its calculation. The error in the estimate of the mean, called the standard error, is given by $S/\sqrt{n}$ where S is the estimate of the standard deviation of the sample and n is the number of measurements.

Reasonable Guess of Measurement Error

For normal laboratory work in an educational setting, it is not unusual for the class to adopt a reasonable guess approach to the measurement error to replace the standard error value. In that approach, measurements which are clearly defective in some fashion are repeated and the unreasonable measurement discarded without any bias. The remaining values are averaged and the average value is taken as the measured value. An estimate is made of “how close to this average value would be expected” if the measurements were repeated. This estimate is enlarged slightly and used as the reported error value.

Reliability of Physical Measurements
(Summary of the Calculation of Errors)

No measured quantity is ever exact. There are errors due to the limits of ability to read the measuring instrument, the calibration of the measuring instrument, and the inherent variations in the physical quantity being measured. The errors shall be presented using the following rules.

1. When a standard or **True value** is known to be high degree of accuracy, designated as **T,** the error is expressed as percent error. The result of a single set of measurements should be presented as

   M ± Percent Error

   where *M* represents the mean value of several trials and the percent error is given by

   Percent Error = [(M – T) / (T)] x 100%

2. When a **standard value is not known**, and a formal analysis is desired, the results of a series of measurements on a single quantity should formally be presented as

   M ± Standard Error where the standard error is

$$S.E. = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n(n-1)}},\ and\ \ M = \frac{\sum x_i}{n}$$

   where x_i is an individual measurement and $s^2 = \sum \frac{(x-\bar{x})^2}{(n-1)}$ is the average or mean value.

3. When a standard value is not known, and a formal error analysis is not warranted, the results of a series of measurements on a single quantity may be presented as

   M ± Estimated Error

   where an estimate is made of "how close to this Mean (average value) would be expected" if the measurements were repeated. This estimate is enlarged slightly and used as the reported error value.

4. When the final result is computed through addition or subtraction of measured quantities, the absolute errors add.

5. When the final result is computed through multiplication or division of measured quantities, the relative percent errors add. For example, finding the area of a table by measuring the sides.

